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Flow control through use of heterogeneous porous media: "Smart" passive thermo-insulating materials

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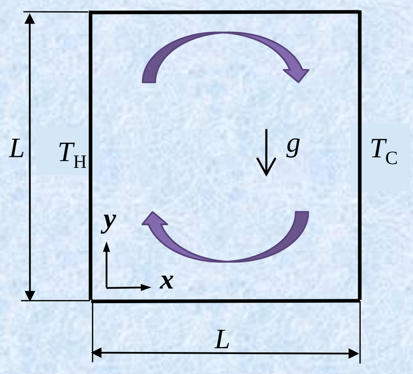
INTRODUCTION

A concept of "smart" thermally insulating materials intelligently adapted for the specified engineering configuration is established and extensively validated. The thermal insulation is achieved by local suppression of the momentum of the confined natural convection flow in the most critical regions as determined by the linear stability analysis of the flow in the presence of implants of heterogeneous porous media. The implants are modelled by unconnected packed beds of equi-sized cylinders. The concept is based on the mesoscale approach for which the non-slip boundary conditions in the vicinity of the packed beds are explicitly imposed by utilizing the immersed boundary (IB) method. Two different patterns of the "smart" porous media materials are established and their thermal insulation properties are quantified. It is shown that the optimized implants of heterogeneous porous media, occupying approximately only 5% of the overall volume, can drastically delay the steady-unsteady transition of the 2D natural convection flow in square differentially heated cavity with thermally perfectly conducting horizontal walls. In addition it is demonstrated that the implants allow to achieve a more than twofold decrease of heat flux rate through the cubic differentially heated cavity with all thermally perfectly conducting lateral walls.

GEOMETRY AND NUMERICAL MODEL

Geometry



Governing equations

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla p + \sqrt{\frac{1}{\mathrm{Gr}}}\nabla^2\boldsymbol{u} + \theta\vec{e}_z$$

$$\frac{\partial \theta}{\partial t} + (\boldsymbol{u}\cdot\nabla)\theta = \frac{1}{\mathrm{Pr}\sqrt{\mathrm{Gr}}}\nabla^2\theta$$

Boundary conditions

Non-slip velocities on all boundaries

Either perfectly conducting or insulated lateral walls

No boundary conditions for pressure

IMMERSED BOUNDARY

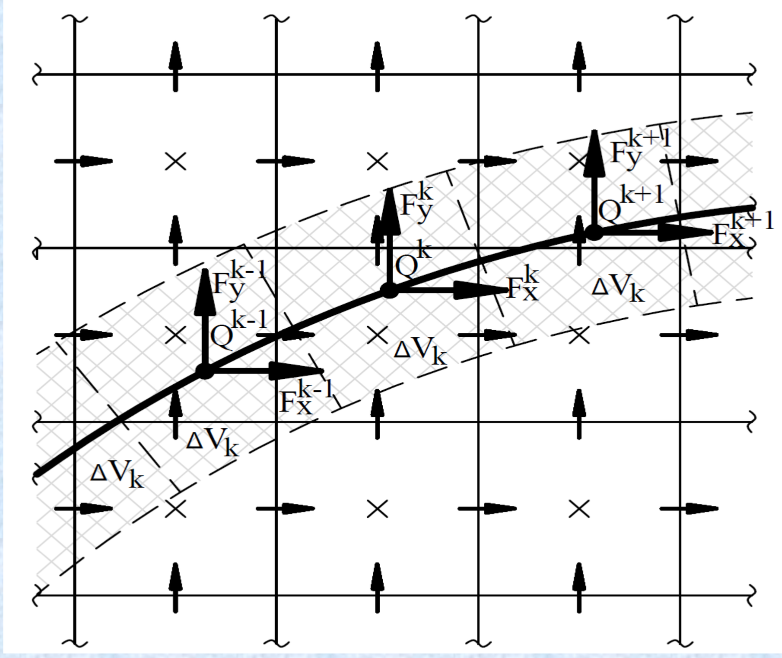
$$\mathbf{A} = |u_x'|^2 + |u_y'|^2$$

Regularization and Interpolation

$$(\boldsymbol{f}_i, q_i) = \Delta x^2 \sum_k (\boldsymbol{F}_k, Q_k) \cdot d(X_k - x_i) \cdot d(Y_k - y_i)$$

$$(\boldsymbol{U}_k, \Theta_k) = \Delta x^2 \sum_i (\boldsymbol{u}_i, \theta_i) \cdot d(x_i - X_k) \cdot d(y_i - Y_k)$$

$$d(r) = \begin{cases} \frac{1}{6\Delta r}\left[5 - 3\frac{|r|}{\Delta r} - \sqrt{-3\left(1 - \frac{|r|}{\Delta r}\right)^2 + 1}\right] & \text{for } 0.5\Delta r \le |r| \le 1.5\Delta r \\ \frac{1}{3\Delta r}\left[1 + \sqrt{-3\left(\frac{|r|}{\Delta r}\right)^2 + 1}\right] & \text{for } |r| \le 0.5\Delta r \\ 0 & \text{otherwise} \end{cases}$$



$$\Delta x = \Delta y \approx \Delta S$$

Block-matrix form

$$\begin{bmatrix} H_u & 0 & 0 & -\nabla_p^x & R_{F_x} & 0 & 0 \\ 0 & H_v & \vec{e_y} & -\nabla_p^y & 0 & R_{F_y} & 0 \\ 0 & 0 & H_\theta & 0 & 0 & 0 & R_Q \\ \nabla_u^x & \nabla_v^y & 0 & 0 & 0 & 0 & 0 \\ I_u & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_v & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_\theta & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u^{n+1} \\ v^{n+1} \\ \theta^{n+1} \\ p \\ F_x \\ F_y \\ Q \end{bmatrix} = - \begin{bmatrix} RHS_u^{n-1,n} \\ RHS_v^{n-1,n} \\ RHS_\theta^{n-1,n} \\ 0 \\ U_b \\ V_b \\ \Theta \end{bmatrix}$$

R – Regularization

I – Interpolation

LINEAR STABILITY

The key idea is to locally suppress the most energetic regions of convective flow, formally revealed by the linear stability analysis of two dimensional convective flow. Due to similar flow behavior in 2D and 3D differentialy heated cavity the regions obtaind for a 2D cavity can be used for 3D cavity.

The linear stability eigneproblem of flow in presence of immersed body is formulated by assuming infinitesimally small perturbations in the form of $\{\tilde{u}(x,y), \tilde{\theta}(x,y), \tilde{p}(x,y), \tilde{\boldsymbol{F}}(x,y), \tilde{Q}(x,y)\}e^{\lambda t}$ around the steady state flow $\boldsymbol{U}, \Theta, p, \boldsymbol{F}, Q$ as follows

$$\lambda\tilde{u} = -(\boldsymbol{U}\cdot\nabla)\tilde{u} - (\tilde{u}\cdot\nabla)\boldsymbol{U} - \nabla\tilde{p} + Gr^{-0.5}\nabla^2\tilde{\boldsymbol{u}} - \tilde{\theta}\vec{e_z} - R_{\tilde{\boldsymbol{F}}}$$

$$\lambda\tilde{\theta} = -(\boldsymbol{U}\cdot\nabla)\tilde{\theta} - (\tilde{u}\cdot\nabla)\Theta + Pr^{-1}Gr^{-0.5}\nabla^2\tilde{\theta} + R_{\tilde{Q}}$$

$$\nabla\cdot\tilde{\boldsymbol{u}} = 0$$

$$I(\tilde{\boldsymbol{u}}) = 0$$

$$I(\tilde{\theta}) = 0$$

solved in a shift-invert mode

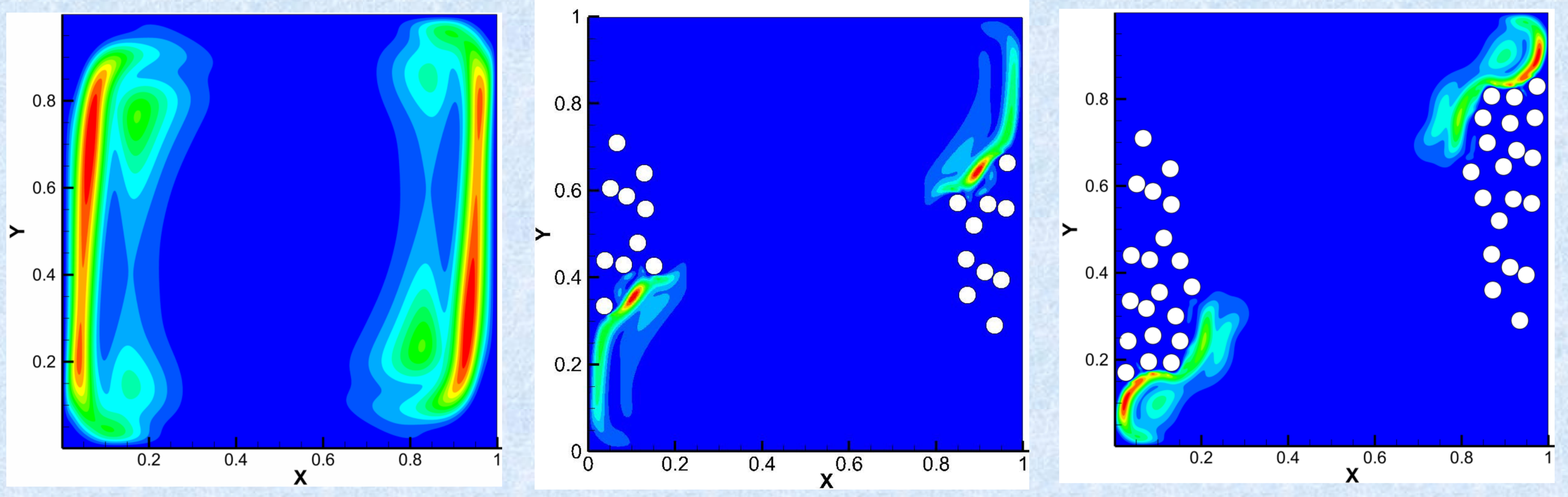
σ- complex shift

$$\lambda\boldsymbol{B}\begin{bmatrix}\tilde{\boldsymbol{u}}\\ \tilde{\theta}\\ \tilde{p}\\ \tilde{\boldsymbol{F}}\\ \tilde{Q}\end{bmatrix} = \boldsymbol{J}\begin{bmatrix}\tilde{\boldsymbol{u}}\\ \tilde{\theta}\\ \tilde{p}\\ \tilde{\boldsymbol{F}}\\ \tilde{Q}\end{bmatrix} \qquad (\boldsymbol{J} - \boldsymbol{\sigma}\boldsymbol{B})^{-1}\boldsymbol{B}\begin{bmatrix}\tilde{\boldsymbol{u}}\\ \tilde{\theta}\\ \tilde{p}\\ \tilde{\boldsymbol{F}}\\ \tilde{Q}\end{bmatrix} = \mu\begin{bmatrix}\tilde{\boldsymbol{u}}\\ \tilde{\theta}\\ \tilde{p}\\ \tilde{\boldsymbol{F}}\\ \tilde{Q}\end{bmatrix} \qquad \mu = \frac{1}{\lambda - \sigma}$$

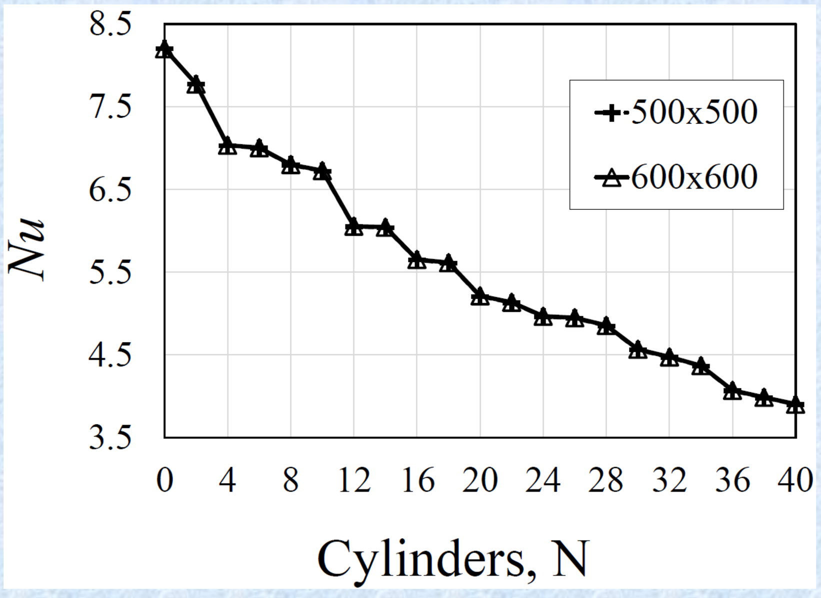
THE MOST ENERGETICALLY ACTIVE REGIONS

Contours of the criterion $\mathbf{A} = |u_x'|^2 + |u_y'|^2$, where $|u_x'|$ and $|u_y'|$ are the absolute values of perturbations of the velocity components.

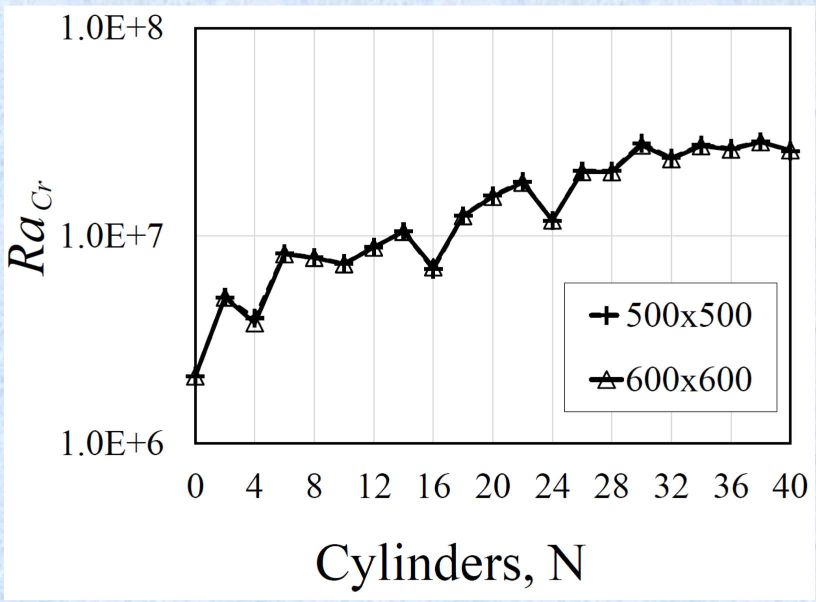
Differentially heated 2D domain



no obstacles 20 obstacles 40 obstacles



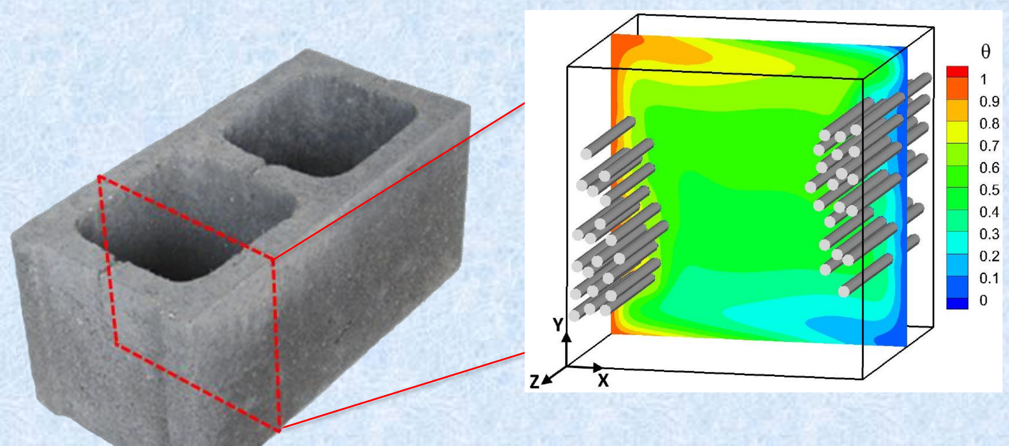
Nu values as function of number of obstacles



Ra_{Cr} values as function of number of obstacles

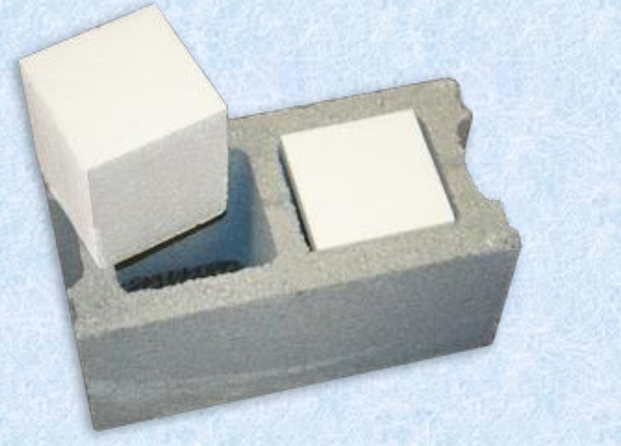
PRACTICAL APPLICATION

"Smart" passive thermo-insulating



Existing Solution

Placement of insulation material into a cavity.



CONCLUSION

The concept of design of "smart" thermally insulating materials was established. The location of each cylinder was determined by an iterative procedure basing on the linear stability analysis of the flow fields. The affect of both optimization criteria **A** and **B** related to perturbation of the velocity and the temperature fields, respectively on insulation properties differentially heated cavities has been investigated. It was found that for the given value of Ra the implants of porous media by utilizing criterion **A** can decrease the overall heat flux rate by the factor of 2. In contrast the implants of criterion **B** can only decrease the heat flux rate by 30%. Implants of porous media designed by both criteria delay the transition to unsteadiness natural convection flow, which is reflected by increasing the critical Ra_{Cr} value by an order of magnitude when using the ultimate pattern consisting of 40 cylinders.